

Lecture 28

Cyclic Processes

We have noted that the industrial revolution began when people first started to extract useful work from enthalpy changes rather than from changes in potential or kinetic energies. The design of such a process involves selecting a working fluid (such as water), providing a way to transfer heat to the fluid to increase its enthalpy, and then allowing the fluid to expand so that useful work is done. Typically, the work would rotate a shaft that would drive a pump, rotate a coil in an electric generator, or turn a screw to propel a ship. The earliest such devices—steam engines—were notoriously inefficient; usually, no more than 2% of the heat added to the fluid would be extracted as useful work. This motivates obvious questions about such devices: How can the efficiency be improved? Is there any limit to the efficiency; that is, is there a limit to the fraction of heat that can be converted into useful work? In particular, can all the heat added to the working fluid be extracted as useful work?

It is not immediately obvious what the answer to this last question might be. After all, it is easy to do the inverse; that is, we can readily convert all the work from a process into heat. For example, you can start your car's engine and let it sit idling in the driveway. All the work being done by the pistons to rotate the crankshaft would be converted into heat that is expelled to the surroundings through the radiator and engine casing. So $W \rightarrow Q$ is easy. Further, the first law ($\Delta U = Q + W$) implies symmetry between heat and work: many changes in U can be produced by only transferring heat, or by only doing work, or by many different combinations of the two. Moreover, Joule could have gathered experimental evidence for conservation of energy by starting with workfree experiments rather than adiabatic ones. Given this apparent symmetry and the fact that $W \rightarrow Q$ is easy, it *seems* likely that a 100% conversion of $Q \rightarrow W$ is at least possible, if not easy. These are the kinds of questions that will occupy us for the next few lectures.

28.1 Some Jargon

Before starting a discussion of conversions of the form $Q \rightarrow \Delta H \rightarrow W$, we introduce and define a few concepts. First, we remind you of a reversible change, which was introduced in Lecture 13.

A *reversible change* of state is a model process driven by differential driving forces. Consequently, the system and its surroundings are no more than differentially out of equilibrium throughout the process, and no work is wasted in overcoming friction.

A *cycle* is a multistep process in which the system repeatedly progresses through the same sequence of states. If all steps in the process are reversible, then one cycle can be represented by a closed path on a state diagram. Figure 28.1 shows an example composed of four steps: isobaric expansion from 1 to 2, isometric cooling from 2 to 3, isobaric compression from 3 to 4, and isometric heating from 4 to 1. Around one complete cycle, the change in any property is zero; e.g., in Figure 28.1,

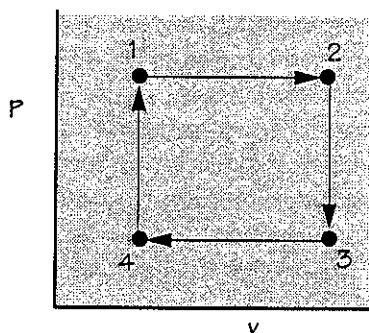
$$\Delta P = \Delta P_{12} + \Delta P_{23} + \Delta P_{34} + \Delta P_{41} = 0 \quad (28.1)$$

A *reservoir* is a huge system that is so massive that its intensive properties (such as T and P) do not change when it exchanges energy with another (smaller) system. Examples of reservoirs include oceans, lakes, and the earth's atmosphere.

A *heat engine* is a cyclic device that takes heat from a reservoir, converts some of that heat into useful work, and expels the remaining heat to another reservoir. A schematic is shown in Figure 28.2. The *thermal efficiency* of a heat engine is the fraction of the heat that is converted into useful work,

$$\eta = \frac{|W|}{Q_{\text{in}}} \quad (28.2)$$

Figure 28.1 Example of a cyclic process in which a working fluid continuously progresses through the sequence of states from $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$.



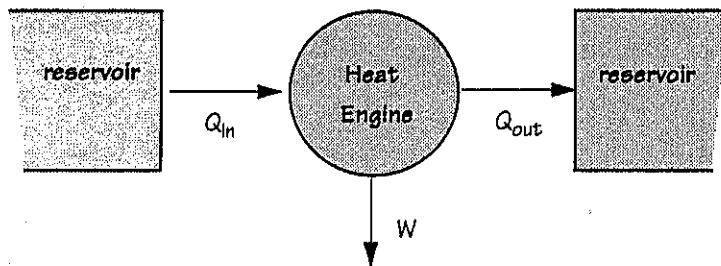


Figure 28.2 Abstract schematic of a heat engine, which takes heat from a reservoir, converts some of it to work, and expels the rest to another reservoir.

28.2 Example of a Heat Engine

An abstract schematic of a heat engine appears in Figure 28.2, but a realistic example is a steam power plant, shown in Figure 28.3. Such a plant uses water as the working fluid; as the water flows around a cycle, it encounters four devices: a boiler, a turbine, a condenser, and a pump. The boiler takes heat from a reservoir to convert water into high-pressure, high-temperature steam. The heat reservoir might be a coal-fired furnace or a nuclear reactor. Steam from the boiler expands onto blades in the turbine, doing work by rotating a shaft. Low-pressure steam in the turbine exhaust is condensed back to liquid in the condenser. The heat removed from the steam is expelled to a low-temperature reservoir, such as a lake or river. From the condenser, the water is pumped back to the boiler to begin a new cycle.

Note that the boiler, turbine, condenser, and pump are each open systems, but the entire heat engine (enclosed by the boundary shown in Figure 28.3) is closed—only energy and not mass crosses its boundary. The first law for the heat engine is therefore

$$\Delta U = Q + W \quad (28.3)$$

where Q is the net heat effect

$$Q = Q_{\text{in}} - |Q_{\text{out}}| \quad (28.4)$$

and W is the net work effect. Let W_t be the work done by the steam at the turbine and let W_p be the work done by the pump on the condensate. Then the net work effect is

$$W = W_p - |W_t| \quad (28.5)$$

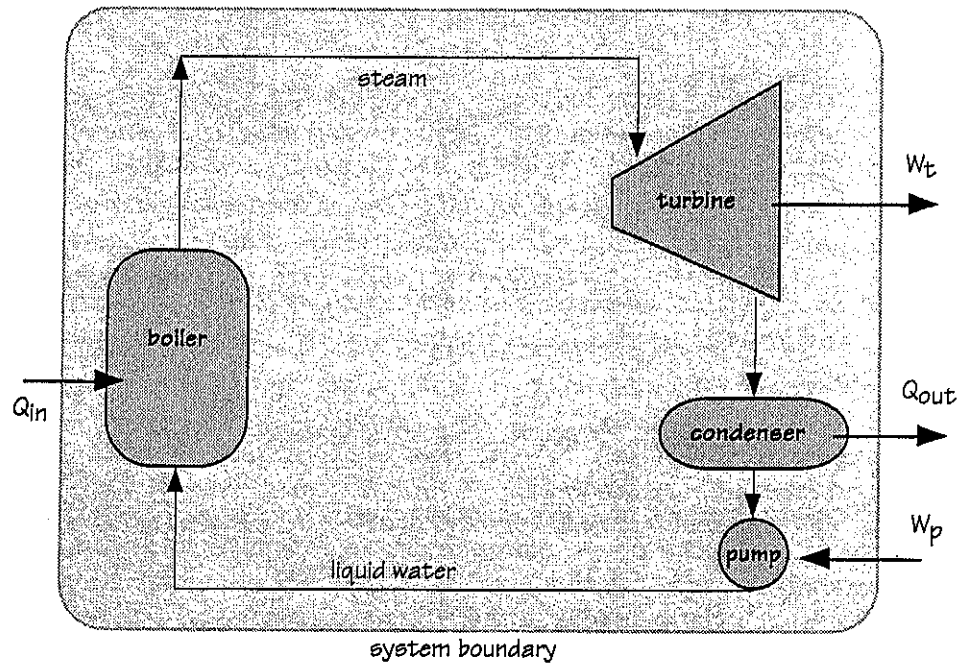


Figure 28.3 Schematic of a steam power plant as a representative example of a heat engine.

The work W_p required to drive the pump is small or even negligible compared to the work W_t produced by the turbine. Around one cycle, $\Delta U = 0$; then (28.3) combined with (28.4) yields

$$W = |Q_{\text{out}}| - Q_{\text{in}} < 0 \quad (28.6)$$

If the net effect of the heat engine is to produce useful work at the turbine, then (28.6) requires that more heat be added at the boiler than is removed at the condenser.

Substituting (28.6) into (28.2) gives the thermal efficiency solely in terms of the amounts of heat transferred,

$$\eta = \frac{Q_{\text{in}} - |Q_{\text{out}}|}{Q_{\text{in}}} \quad (28.7)$$

This expression helps us clarify our questions about the efficiency. To improve the efficiency, we should try to reduce the amount of heat removed at the condenser or increase the amount of heat added at the boiler or both.

28.3 The Carnot Cycle

In designing a heat engine, we must meet at least three important requirements: (1) Heat must be added to the working fluid, (2) The fluid must expand to obtain useful work, (3) The process must close a cycle so that more work can be obtained. Apparently, closing the cycle requires us to do some work on the fluid and remove some heat from it.

Many different processes can meet these requirements, but in the early 1820s, a young Frenchman named Sadi Carnot proposed a particular process that provides additional insight into the questions surrounding the efficiencies of heat engines. Carnot's analysis was published in 1824; it is available in an English translation by Fox [1]. Carnot's analysis established a number of ideas that are crucial to thermodynamics; for example, it was Carnot who invented the idea of a reversible change of state. Carnot's process consists of two turbines and two compressors operating in a four-step cycle:

- A. Reversible isothermal expansion during which some useful work W_A is done and heat Q_{in} is added to the working fluid from a source at a high temperature T_H .
- B. Reversible adiabatic expansion in which more useful work W_B is obtained.
- C. Reversible isothermal compression during which work W_C is done on the fluid and heat Q_{out} is moved to a sink at a low temperature T_L .
- D. Reversible adiabatic compression in which work W_D is done on the fluid to close the cycle.

Note that each step in the Carnot process is reversible. We now apply the PSALMS to obtain the thermal efficiency for the Carnot cycle.

Picture. A schematic for the process appears in Figure 28.4; the corresponding Pv diagram is shown in Figure 28.5. You should ensure that you can connect states 1, 2, 3, and 4 on the Pv diagram to points on the schematic.

System. The working fluid moving through the cyclic process.

Always True. Each step in the process takes place in an open device, so we appeal to the first law for open systems. We use the form from (15.13)

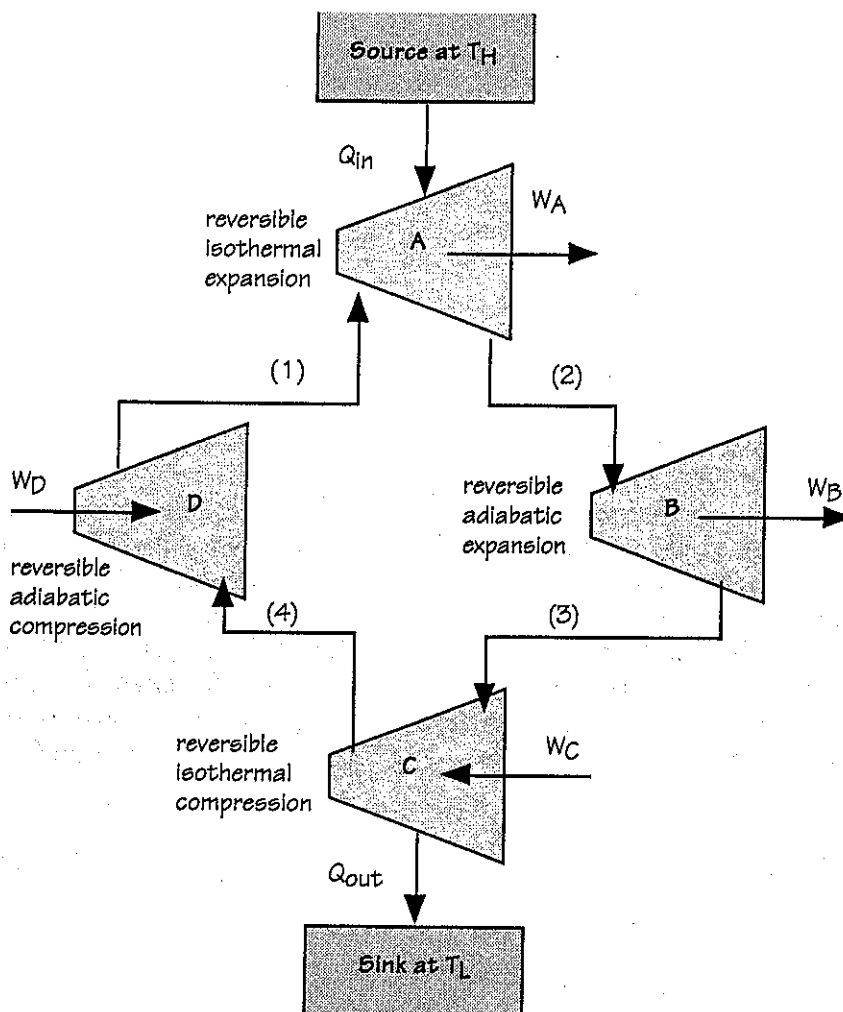


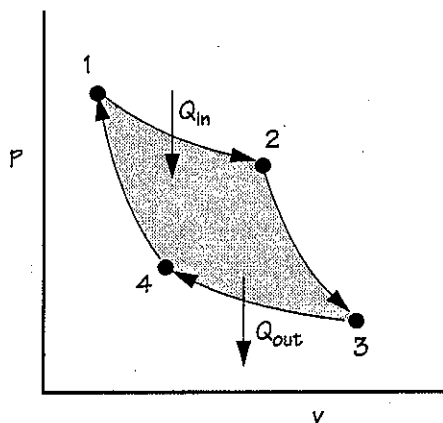
Figure 28.4 Schematic of the Carnot cycle in which heat is taken from a source, work is done by expanding a fluid in turbines A and B, heat is expelled to a sink, and the cycle is closed by doing work on the fluid in compressors C and D.

$$\Delta E_k + \Delta E_p + \Delta U = Q + W \quad (28.8)$$

Model. For the working fluid, we choose an ideal gas with constant heat capacities, since this simplifies the analysis. For the process, each step is reversible.

Strategy. To obtain the efficiency from (28.7), we apply the first law (28.8) to the isothermal steps because those steps have $Q \neq 0$. For each step, we

Figure 28.5 Pv diagram with states of the fluid following the Carnot cycle of Figure 28.4:
 $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$.



assume changes in kinetic and potential energy are negligible, so (28.8) reduces to

$$\Delta U = Q + W \quad (28.9)$$

Step A (reversible isothermal expansion). For an ideal gas, $U = U(T)$, so (28.9) becomes

$$\Delta U_A = Q_{in} + W_A = 0 \quad (28.10)$$

$$Q_{in} = -W_A = \int_{V_1}^{V_2} P dV = RT_H \int_{V_1}^{V_2} \frac{dV}{V} \quad (28.11)$$

$$= RT_H \ln \frac{V_2}{V_1} = RT_H \ln \frac{P_1}{P_2} > 0 \quad (28.12)$$

The temperature of the working fluid along the isotherm is T_H , the temperature of the source, because the expansion is reversible; that is, the driving force for heat transfer is differential.

Step C (reversible isothermal compression). The analysis exactly parallels that just done for step A,

$$\Delta U_C = Q_{out} + W_C = 0 \quad (28.13)$$

$$Q_{out} = -W_C = RT_L \ln \frac{P_3}{P_4} < 0 \quad (28.14)$$

Using (28.12) and (28.14) in (28.7), the efficiency is

$$\eta = \frac{RT_H \ln(P_1/P_2) + RT_L \ln(P_3/P_4)}{RT_H \ln(P_1/P_2)} \quad (28.15)$$

We must now relate the pressure ratio (P_1/P_2) to the ratio (P_3/P_4). To do so, we consider the two adiabatic steps B and D. For an ideal gas with constant heat capacities, the pressures and temperatures along reversible adiabats are related by (17.26); thus,

$$\text{step B:} \quad \frac{T_H}{T_L} = \frac{T_2}{T_3} = \left(\frac{P_2}{P_3}\right)^{(\gamma-1)/\gamma} \quad (28.16)$$

$$\text{step D:} \quad \frac{T_H}{T_L} = \frac{T_1}{T_4} = \left(\frac{P_1}{P_4}\right)^{(\gamma-1)/\gamma} \quad (28.17)$$

where γ is the ratio of the heat capacities, $\gamma = C_p/C_v$. Equating (28.16) and (28.17) yields

$$\frac{P_2}{P_3} = \frac{P_1}{P_4} \quad (28.18)$$

and cross multiplying leaves

$$\frac{P_4}{P_3} = \frac{P_1}{P_2} \quad (28.19)$$

Using this in (28.15), our expression for the efficiency simplifies to

$$\eta_{\text{Carnot}} = \frac{T_H - T_L}{T_H} \quad (28.20)$$

where the temperatures are those of the source and sink. We emphasize that this result is for the Carnot *model* heat engine—it is *not* always true. The always true relations for the thermal efficiency are (28.2) and (28.7). Note that no property of the working fluid appears in the Carnot efficiency. This means that the result is independent of the substance used for the working fluid and that our use of an ideal gas does not restrict the result. The values for the temperatures in (28.20) must be absolute. In practice, the sink is usu-

ally a lake, river, ocean, or the atmosphere, so the temperature T_L is usually ambient. Consequently, the corresponding values for the Carnot efficiency rarely exceed 50%; the efficiency of a real heat engine is considerably less.

Literature Cited

- [1] S. Carnot, *Reflexions on the Motive Power of Fire*, translated and edited by R. Fox, Lilian Barber Press, New York, 1986.

Problems

- 28.1 A submarine is driven by a steam power plant that uses 600 psia, saturated steam from a boiler to drive a turbine that turns the screw. (a) Determine the Carnot efficiency of this plant when the submarine is running on the surface of the South Pacific where the ocean temperature is 80°F. (b) Repeat the calculation for the submarine running submerged under the polar ice cap, where the water is about 35°F. Conclusion?
- 28.2 Sketch a Pv diagram for an Carnot cycle that uses an ideal gas as the working fluid. On your sketch locate each of the following: (a) the temperature of the source T_H , (b) the temperature of the sink T_L , (c) the total work of expansion in one cycle, (d) the total work of compression in one cycle, (e) the net work done by the engine in one cycle.
- 28.3 A heat engine operates between a source at 250°C and a sink at 25°C. (a) What is the Carnot efficiency? (b) The actual efficiency of this engine is 30% of the Carnot efficiency. If the engine takes in 103 kJ of heat per hour from the source, what is its power output (kW)?
- 28.4 Pure nitrogen is used as the working fluid in a heat engine that undergoes the following three-step cycle:
- step A: Irreversible isometric heating from 25°C and 1 bar to 5 bar,
 - step B: Irreversible adiabatic expansion to 1 bar,
 - step C: Reversible isobaric cooling back to the initial state.
- (a) Determine the thermal efficiency for this heat engine.
- (b) Sketch this cycle on a Pv diagram and locate the net work on your sketch.

Lecture 29

Can $\eta = 1$?

The thermal efficiency of a heat engine is defined to be the fraction of added heat that is converted into useful work,

$$\eta = \frac{|W|}{Q_{\text{in}}} = \frac{Q_{\text{in}} - |Q_{\text{out}}|}{Q_{\text{in}}} \quad (29.1)$$

For the special case of the heat engine operating in a Carnot cycle, this efficiency depends only on the temperatures of the heat source T_H and the heat sink T_L ,

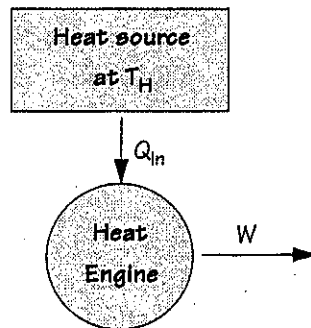
$$\eta_{\text{Carnot}} = \frac{T_H - T_L}{T_H} \quad (29.2)$$

We want the efficiency to be as large as possible, so in this lecture we determine whether there is any limit on the efficiency. For example, can we design a heat engine that operates at 100% efficiency ($\eta = 1$)? This means all the heat added to the engine would be converted into useful work. If this is not possible, then we ask, What is the upper bound on η ?

29.1 Repose the Question

The question is whether we can operate a heat engine so that $\eta = 1$. This means the engine would have to run without expelling any heat to its surroundings. That is, we would need $Q_{\text{out}} = 0$, and then (29.1) would reduce to $\eta = Q_{\text{in}}/Q_{\text{in}} = 1$. By the first law, all the heat provided to the engine would be converted to useful work: $W = Q_{\text{in}}$. A schematic of such a (hypothetical) device is shown in Figure 29.1.

Figure 29.1 A heat engine operating at 100% efficiency would expel no heat to its surroundings. Our problem is to decide whether or not such an engine is possible.



But we also have another constraint: we not only need $Q_{\text{out}} = 0$, but we also want the heat engine to operate in a cycle, so the engine continuously provides work. Two such hypothetical cycles are shown on the Pv diagrams in Figure 29.2. In cycle **A**, the working fluid undergoes a workfree heating from 1 to 2. From state 2 it expands reversibly and adiabatically to 3, doing useful work. These two steps are certainly possible. The issue now is whether we can close the cycle; that is, can we adiabatically ($Q_{\text{out}} = 0$) compress the fluid from 3 back to the initial state 1?

Cycle **B** in Figure 29.2 is similar to cycle **A**, except we consider a reversible isothermal expansion from 2 to 4. (State 4 in cycle **B** would differ from state 3 in cycle **A**.) During the isothermal expansion we would add more heat to keep T constant. The question is still whether we can close the cycle by adiabatically compressing the fluid from 4 back to 1. In short, the question of whether we can have $\eta = 1$ can be rephrased as this:

Are there any constraints on adiabatic processes?

We first consider reversible adiabatic processes and then irreversible ones.

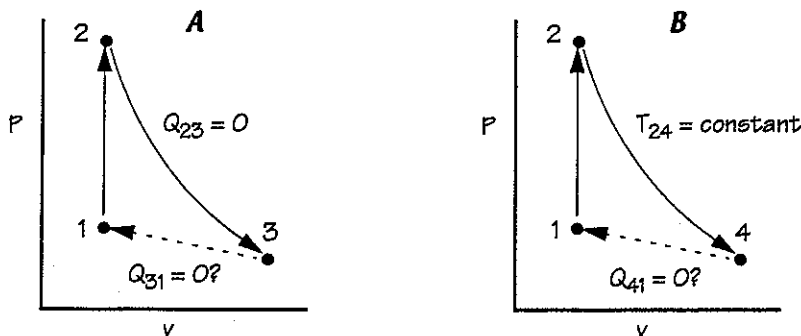


Figure 29.2 Two hypothetical processes for heat engines that would operate at 100% efficiency. In each case the issue is whether the cycle can be closed without removing heat.

29.2 Reversible Adiabatic Processes

Consider a working fluid in an initial state 1 identified by values for its volume and internal energy (V_1, U_1), as in Figure 29.3. From this initial state, we reversibly and adiabatically compress the fluid to a smaller volume V_2 . The issue is whether the reversible adiabatic path from 1 to 2 is unique. The compression requires a particular amount of work, given by

$$W = - \int_{V_1}^{V_2} P dV = \text{a number} \quad (29.3)$$

Given a model for $P(V, T)$, we could compute this amount of work.

By the first law for an adiabatic process, this work changes the internal energy of the fluid,

$$\Delta U = W_{\text{rev}} \quad \text{adiabatic} \quad (29.4)$$

Therefore, ΔU is a particular value, and so, from the initial state with U_1 , the final value of the internal energy is unique,

$$U_2 = U_1 + \Delta U \quad (29.5)$$

The final values V_2 and U_2 locate *one* state on the UV diagram, as in Figure 29.3.

We can repeat this argument for reversible adiabatic compressions from 1 to *any* $V_2 < V_1$; it also applies to reversible adiabatic expansions from 1 to *any* $V_3 > V_1$. Thus from state 1, the final states (V_f, U_f) accessible by reversible adiabatic processes trace a *unique* curve on a UV diagram. Further, from

$$dU = dW_{\text{rev}} = -PdV \quad \text{adiabatic} \quad (29.6)$$

the slope of the reversible adiabat must be negative,

Figure 29.3 From initial state 1, a reversible adiabatic compression to V_2 leaves the system in a final state that must have internal energy U_2 .

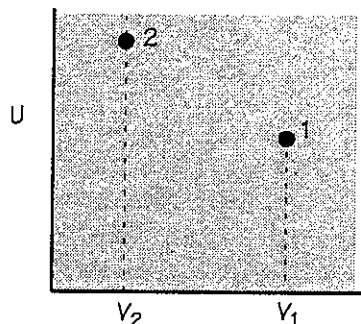
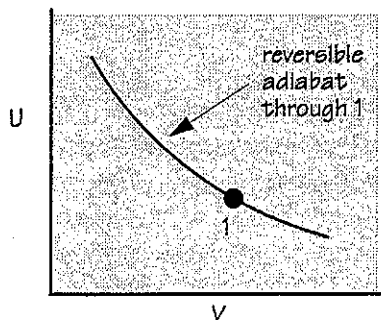


Figure 29.4 Through any state 1 of a single homogeneous phase there passes only *one* reversible adiabat.



$$\left(\frac{\partial U}{\partial V}\right)_{Q_{\text{rev}}=0} = -P \quad (29.7)$$

Such a reversible adiabat is shown schematically in Figure 29.4.

The consequences of this argument are important: through any state on a phase diagram there passes only *one* reversible adiabat. So from a given initial state, only a limited range of states can be reached by reversible adiabatic processes. In cycle *A* of Figure 29.2, this means that the two paths through state 3 cannot both be reversible adiabats. If the path $2 \rightarrow 3$ is the reversible adiabat, then 1 cannot be reached from 3 by a reversible adiabatic compression. However, this does not yet preclude *irreversible* adiabatic compressions; we consider that possibility next.

29.3 Irreversible Adiabatic Processes

Consider, again, a working fluid at an initial state identified by values U_1 and V_1 . Now we compress the fluid adiabatically and irreversibly to $V_2 < V_1$. In any irreversible compression, some amount of work is needed to overcome friction, so the total work required is more than would be needed for the same process performed reversibly,

$$W_{\text{irr}} > W_{\text{rev}} > 0 \quad (29.8)$$

The difference between these two values is the lost work, which is dissipated as heat at the system boundary. But here we are considering situations in which the boundary is insulated ($Q = 0$), so the frictional heat cannot pass to the surroundings; instead, it passes into the system, where it increases the fluid's internal energy. This means the value of U attained by the irreversible compression must be larger than that attained by a reversible compression to the same V_2 ,

$$U_{\text{irr}}(V_2) > U_{\text{rev}}(V_2) \quad \text{compression} \quad (29.9)$$

The final state at V_2 must lie above the reversible adiabat through (U_1, V_1) .

This same argument applies for irreversible adiabatic expansions from the initial state (U_1, V_1) . During an irreversible expansion, friction again creates heat at the boundary, and this heat again increases U relative to its final value for a reversible expansion. If the final volume is to be $V_3 > V_1$, then the form (29.9) still applies; i.e.,

$$U_{\text{irr}}(V_3) > U_{\text{rev}}(V_3) \quad \text{expansion} \quad (29.10)$$

These arguments impose a severe constraint on adiabatic processes: from an initial state, irreversible adiabatic processes can move the fluid *only* to states that lie above the reversible adiabat on a U - V diagram. States below the reversible adiabat can be reached only by processes that involve heat transfer from the system. These remarks are illustrated in Figure 29.5. Consequently, neither cycle proposed in Figure 29.2 is possible. More generally, we cannot close a thermodynamic cycle without expelling some heat to the surroundings. Thus, every heat engine must have $|Q_{\text{out}}| > 0$, and therefore we cannot build a heat engine that operates at 100% efficiency,

$$\eta < 1 \quad (29.11)$$

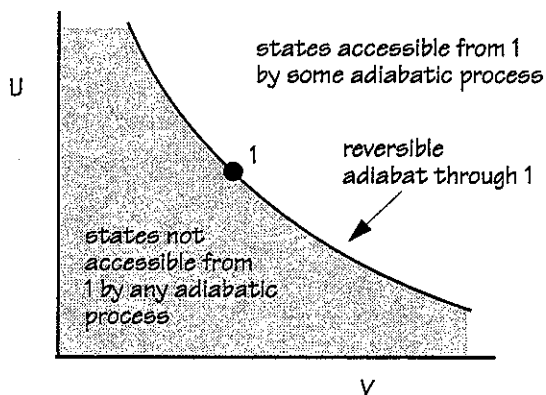


Figure 29.5 From a given initial state 1, only states above the reversible adiabat through 1 can be reached by some adiabatic process. States below the reversible adiabat can only be reached by processes that remove heat from the system.

29.4 A New Property

Since no heat engine can operate at 100% thermal efficiency, the obvious next question is, What is the maximum efficiency attainable by any heat engine? We will answer this question in § 29.5, but the resolution of that question is simplified if we can appeal to a property that we have not yet encountered. We introduce that new property here.

Recall that a property is a state function—a quantity that forms an exact differential. To have a notation, let us use the symbol S for the new property. So we require that dS be exact; this means that a change of state will be accompanied by a change in S that is calculable by

$$\Delta S = S_2 - S_1 \quad (29.12)$$

This will apply regardless of the process used to reach 2 from 1.

As a second attribute of S , let us appeal to reversible adiabats. In § 29.2 we showed that, through any state 1, there passes only one reversible adiabat (see Figure 29.4). So, let us require that the new property be constant along reversible adiabats,

$$\Delta S = 0 \quad \text{reversible adiabats} \quad (29.13)$$

Each reversible adiabat is unique, so we can assign to each a unique value of S . This means that dS should be proportional to dQ_{rev} ,

$$dS \propto dQ_{\text{rev}} \quad (29.14)$$

But now we seem to have an inconsistency because we know dQ is generally inexact, but we want dS to always be exact.

We can resolve this apparent inconsistency by recalling from the calculus that many inexact differentials can be made exact by finding an integrating factor. Perhaps there is a factor λ that converts dQ_{rev} into an exact differential? If so, we would have

$$dS = \frac{dQ_{\text{rev}}}{\lambda} \quad (29.15)$$

Why should we believe such a factor exists? First, there is an argument based on analogy with the reversible work. Recall the reversible work is given by

$$dW_{\text{rev}} = -P dV \quad (29.16)$$

Hence we can write

$$dV = \frac{-dW_{\text{rev}}}{P} \quad (29.17)$$

and we know dV is exact (V is a property). Thus, $(-1/P)$ is an integrating factor that converts dW_{rev} into an exact differential. We expect there to be some other integrating factor that does the same for dQ_{rev} . Second, proof of the existence of an integrating factor for dQ_{rev} has been given by Carathéodory; it is explained by Chandrasekhar [1] and Kestin [2]. Third, experimental evidence confirms the existence of λ .

Now if any λ does exist, then the calculus assures us that, in fact, an infinite number of integrating factors exist. We have an embarrassment of riches—a huge number of possibilities are available for defining the new property. The simplest possibility is to choose λ to be the absolute temperature. So, we define the new property by

$$dS \equiv \frac{dQ_{\text{rev}}}{T} \quad \text{always true} \quad (29.18)$$

This quantity is called the *entropy*; it is an extensive, conceptual, state function. It has dimensions of (energy/temperature). We will explore the entropy more fully in Lecture 31, but for now note two special cases: one is $\Delta S = 0$ along reversible adiabats, as in (29.13), the other is

$$\Delta S = \frac{Q_{\text{rev}}}{T} \quad \text{reversible isotherms} \quad (29.19)$$

29.5 What is the Maximum Efficiency for a Heat Engine?

Consider the heat engine shown in Figure 29.6. This engine takes heat Q_{in} from a high-temperature source at T_{H} , supplies useful work W to a work reservoir, and expels heat Q_{out} to a low-temperature sink at T_{L} . We want to find an expression for the maximum efficiency of this engine. As the system in this analysis, we take the entire contents of the figure: source, sink, heat engine, and work reservoir. The boundary between this system and its surroundings is closed and insulated. The efficiency for the heat engine is given by (29.1), which we write as

(29.17)

n integrating
ct there to be
ond, proof of
iven by Car-
a [2]. Third,

n fact, an infi-
arrassment of
ning the new
lute tempera-

(29.18)

ial, state func-
explore the
cases: one is

(29.19)

Engine?

ngine takes heat
ork W to a work
 T_L . We want to
. As the system
ource, sink, heat
tem and its sur-
: heat engine is

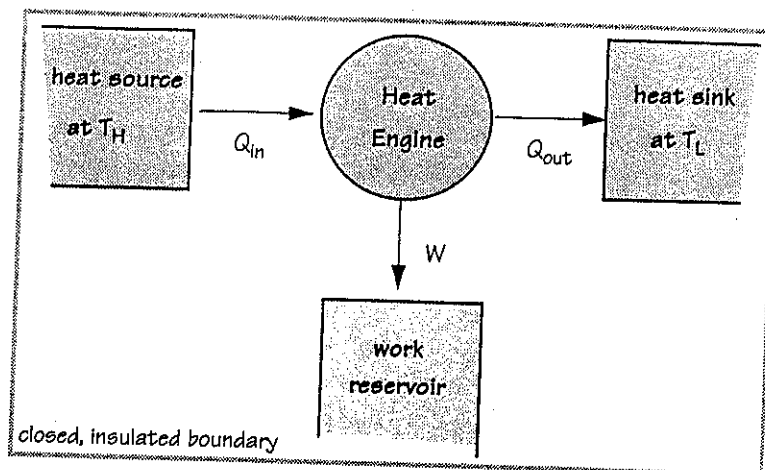


Figure 29.6 Schematic of a heat engine that converts some heat from a source into useful work and expels the remaining heat to a sink.

$$\eta = 1 - \frac{|Q_{out}|}{Q_{in}} \quad (29.20)$$

We seek an expression for the ratio Q_{out}/Q_{in} that maximizes η . To obtain the maximum efficiency, we already know that every step in the cycle must be reversible: all energy transfers to and from the heat engine must be reversible.

Consider the change in the total entropy of the system through one cycle of the heat engine:

$$\Delta S_{tot} = \Delta S_{so} + \Delta S_{si} + \Delta S_{he} + \Delta S_{wr} = 0 \quad (29.21)$$

The subscripts are to be interpreted as so = heat source, si = heat sink, he = heat engine, and wr = work reservoir. This change in total entropy is zero because all steps are reversible and the system boundary is insulated: the entire process is reversible and adiabatic.

In addition, we have $\Delta S_{he} = 0$ around one cycle, and $\Delta S_{wr} = 0$ because the process supplying work to the work reservoir is itself reversible and adiabatic. Therefore, (29.21) reduces to

$$\Delta S_{so} + \Delta S_{si} = 0 \quad (29.22)$$

The reservoirs used for the source and sink are so huge that their temperatures are not affected by the heat transfers to and from the engine; that is, those steps are reversible and isothermal. So by (29.19), we write (29.22) as

$$\frac{Q_{\text{in}}}{T_H} - \frac{|Q_{\text{out}}|}{T_L} = 0 \quad (29.23)$$

Hence, the ratio of the amounts of heat is the same as the ratio of the absolute temperatures of the source and sink,

$$\frac{|Q_{\text{out}}|}{Q_{\text{in}}} = \frac{T_L}{T_H} \quad (29.24)$$

Using this in (29.20) gives the maximum efficiency as

$$\eta_{\text{max}} = 1 - \frac{T_L}{T_H} = \frac{T_H - T_L}{T_H} \quad (29.25)$$

But by (29.2), this is the Carnot efficiency! The Carnot cycle provides the maximum thermal efficiency that can be obtained from a heat engine operating between a given heat source and a given heat sink. Equation (29.25) is sometimes called the *maximum-work* theorem, but this is a misnomer: a Carnot cycle always maximizes the thermal efficiency, but it may not maximize the amount of work produced. In Lecture 34 we will show that an ideal Rankine cycle may produce more useful work than a Carnot cycle, even though the Rankine cycle has a lower thermal efficiency.

Literature Cited

- [1] S. Chandrasekhar, *An Introduction to Stellar Structure*, Dover, New York, 1958, Ch. 1.
- [2] J. Kestin, *A Course in Thermodynamics*, vol. 1, Revised Printing, Hemisphere Publishing, New York, 1979.

Problems

- 29.1 List at least three thermodynamic quantities that have dimensions of (energy/temperature mole).
- 29.2 Sketch a schematic Pv diagram for a Carnot cycle. Let numerals 1, 2, 3, 4 identify states of the working fluid at the ends of steps in the cycle. Let

heat be added in step 1→2 and let heat be removed in step 3→4. For each of the four steps, show whether $\Delta S > 0$, or $\Delta S = 0$, or $\Delta S < 0$.

- 29.3 Consider an ideal gas that has $c_p = 7R/2$ and let state 1 be at 25°C and 1 bar. (a) On a Pv diagram for this substance, show state 1 along with the isotherm and the reversible adiabat that pass through 1. (b) Add to your diagram the lines $U = \text{constant}$ and $S = \text{constant}$ that also pass through 1.

- 29.4 A heat engine, operating at 50% of its maximum thermal efficiency, takes 4200 kJ/min of heat from a source at 350°C and expels heat to a sink at 30°C. (a) What is the useful power output (kW) from this engine? (b) What is the rate at which heat is being expelled to the sink?

- 29.5 (a) Sketch a schematic UV diagram, like that in Figure 29.5, for an ideal gas that has $c_p = 7R/2$. Let state 1 be at 25°C and 1 bar. Draw the reversible adiabat through 1; label the regions of accessible and inaccessible adiabatic states. (b) Now transform your sketch to a PV diagram for the same substance. Show the same features on the PV diagram as are on your UV diagram.

- 29.6 One kg of steam, initially at 200°C and 10 bar, undergoes a reversible isothermal expansion to 1 bar. Steam tables give the following for these two states, both at 200°C:

P (bar)	v (cc/g)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg K)
1	2172	2658	2875	7.834
10	206.0	2622	2828	6.694

- (a) Determine the amount of heat transferred during this process. Is heat added to or removed from the steam?
(b) Determine the amount of work done.

- 29.7 One kg of steam, initially at 600°C and 10 bar, expands reversibly and adiabatically to 1 bar. Steam tables give the following:

T (°C)	P (bar)	v (cc/g)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg K)
600	10	401	3297	3698	8.029
240	1	2359	2718	2954	7.995
280	1	2546	2780	3034	8.144

- (a) Determine the temperature at the final state.
(b) Determine the amount of work done.

